1. **On tossing a coin 30 times outcomes of head and tail are found as;**

**Head, Head, Tail, Head, Tail, Head, Head, Tail, Tail, Head, Tail, Head, Head, Tail, Tail, Head, Head, Head, Tail , Head, Tail, Head ,Head , Tail, Tail, Head, Tail, Tail, Tail, Head.**

1. **Are outcomes in random order?**
2. **Is coin unbiased?**

**Using 1% level of significance**

Working formula

To test randomness

Problem to test

: Samples are in random order

: Samples are not in random order

Test statistic

Z = ,

Where r = number of run

= + 1

=

P = prob(Z> )

Level of significance = α

Decision

Reject at α level of significance if p< α , accept otherwise

To test unbiasedness

Problem to test

: P = 1/2

: P

Test statistic

Z = ,

Where

n = , use -0.5 if and +0.5 if

p = prob(Z> )

Level of significance = α

Decision

Reject at α level of significance if p< α , accept otherwise

1. **Marks secured by a sample of 32 students in Final examination of Statistics I are found as 43, 52, 34, 56, 28, 12, 46, 38, 10, 51, 49, 38, 46, 24, 36, 44, 38, 46, 49, 27, 35, 41, 11, 23, 35, 42, 52, 49, 20, 35, 43, 37.**
2. **Are samples selected in random order?**
3. **Are marks uniformly distributed? Use Kolmogorov Smirnov test**
4. **Are marks uniformly distributed? Use chi square test**

**Using 5% level of significance.**

Working formula

To test randomness

Find median then to sample values assign symbol say A if sample value less than median and symbol say B if sample value more than median and omit if sample value is equal to median

Problem to test

: Samples are in random order

: Samples are not in random order

Test statistic

Z = ,

Where r = number of run

= + 1

=

P = prob(Z> )

Level of significance = α

Decision

Reject at α level of significance if p< α , accept otherwise

To test uniformity

Problem to test

: Marks are uniformly distributed

: Marks are not uniformly distributed

Kolmogorov Smirnov test

Test statistic

D0 = Max |Fe – F0|

Where Fe = /N , Fo = /N

Critical value

At α level of significance critical value is

Decision

Reject H0 at α level of significance if D0 > Dn, a accept otherwise.

Chi square test

Test statistic

=

Critical value

At α level of significance critical value is Critical value χ²α(k-1)

Decision

Reject H0 at α level of significance if > χ²α(k-1)  accept otherwise.

1. **Following are marks secured by 14 students of section A and 15 students of section B of DWIT in final examination of Digital logic are found as**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Section A** | **34** | **48** | **21** | **52** | **31** | **43** | **29** | **37** | **24** | **52** | **49** | **34** | **40** | **48** |  |
| **Section B** | **11** | **53** | **27** | **38** | **47** | **50** | **26** | **38** | **44** | **33** | **27** | **33** | **41** | **10** | **28** |

**Is median marks of section A and section B identical at 5% level of significance using?**

1. **Median test**
2. **Mann Whiteny U test**
3. **Kolmogorov Smirnov test**

Working formula

Problem to test

Combine n₁ and n₂ such that n = n₁+n₂ and obtain median of n observation. Find number of observations in xᵢ ≤ Md and denote by

a. Find number of observations in yᵢ ≤ Md and denote by b.

Also find the number of observations in xᵢ ˃Md an yᵢ ˃Md and denote by c and d respectively.

|  |  |  |  |
| --- | --- | --- | --- |
|  | No. of obs.≤ Md | No. of obs.˃Md | Total |
| Sample x | a | c | a+c |
| Sample y | b | d | b+d |
| Total | a+b | c+d | N=a+b+c+d |

Test statistic

χ² =  ~ χ²(1)

If any cell frequency is less than 5 then

Corrected χ² = ~ χ²(1)

Critical value

At α level of significance for 1 degree of freedom critical value is χ²α(1)

Decision

Reject H0 at α level of significance if χ² ˃ χ²α(1), accept otherwise.

Using Mann Whitney U test

Combine n₁ and n₂ such that n₁+n₂= n and rank these n observations in ascending order .If two or more observations are equal then assign average rank and is called tied. Sum the ranks of sample of sizes n₁ and n₂ separately to get R₁ and R₂. If two sample sizes are unequal then smaller one is n1. Obtain U1 and U2 as U1 = n1n2 + – R₁ and U2 = n1n2 + – R2.

U0 = min {U1, U2}

Test statistic

Z = σ= it is used even if tied occurs within sample. If tied occurs between samples then standard deviation is corrected as σᵤ =

ti = number of times ith rank repeated between samples.

Critical value

Critical value Ztabulated is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H0 at α level of significance if Z ˃Ztabulated, Accept otherwise.

Using Kolmogorov Smirnov test

Test statistic

D0 = maximum |F(x) – F(y)| , where F(x) = , F(y) =

Critical value

At α level of significance critical value for n1 and n2 is

Dn1, n2, α

Decision

Reject H0 at α level of significance if D0 > D n1, n2, α

Accept otherwise

1. **Following information are obtained from locality related to gender and eye color.**

|  |  |  |
| --- | --- | --- |
| **Person** | **Gender** | **Eye color** |
| **A** | **Male** | **Black** |
| **B** | **Female** | **Black** |
| **C** | **Male** | **Brown** |
| **D** | **Male** | **Black** |
| **E** | **Female** | **Blue** |
| **F** | **Male** | **Brown** |
| **G** | **Female** | **Black** |
| **H** | **Male** | **Black** |
| **J** | **Female** | **Black** |
| **K** | **Female** | **Brown** |
| **L** | **Female** | **Black** |
| **N** | **Male** | **Black** |
| **O** | **Female** | **Blue** |
| **P** | **Female** | **Brown** |
| **Q** | **Male** | **Black** |
| **R** | **Female** | **Black** |
| **S** | **Male** | **Brown** |
| **T** | **Female** | **Black** |
| **U** | **Female** | **Black** |
| **V** | **Male** | **Brown** |

**Is there any association between gender and eye color? Use 5% level of significance.**

Working formula.

Problem to test

H0: Attributes are independent.

H1: Attributes are dependent.

Test statistic

χ² = ΣΣ ~ χ²(r- 1)(c -1)

Eij = ()/N

Critical value

Let α be the level of significance then critical value is χ²α (r– 1)(c– 1)

Decision

Reject H0 at α level of significance if χ² ˃ χ²α (r– 1)(c– 1)

1. **Marks secured by a sample of 15 students of a college in first test and second test of Statistics II are found as**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Student** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** |
| **Test I** | **12** | **7** | **15** | **11** | **17** | **19** | **5** | **13** | **17** | **6** | **9** | **18** | **14** | **10** | **8** |
| **Test II** | **14** | **5** | **17** | **13** | **12** | **18** | **9** | **10** | **18** | **12** | **3** | **14** | **16** | **16** | **8** |

**Is there improvement in marks in test II as compared to test I? Use non parametric test at 5% level of significance**

Working formula

Problem to test

H0: MdI= MdII

H₁: MdI < MdII

Find di = yi – xi or xi – yi for each pair of observations (xi, yi) , i = 1,2,3…….n. Rank di irrespective of sign in ascending order but omit di = 0 . If two or more di are equal then assign the average rank and is called tied. In such case corrected sample size nc = n – t , t is number of tied occurred .Assign sign to the ranks with respect to the sign of di. Sum the ranks of + sign and - sign separately to get S(+) and S(-) respectively. Finally get T = min {S(+) , S(-)}

Test statistic

Z = = ~ N(0, 1) (Here n = nc)

Critical value

Critical value Ztabulated is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H0 at α level of significance if Z ˃Ztabulated, Accept otherwise.

1. **Four diets are fed to 9 cows, each diet for a month and the result of increase(I) and decrease(D) of milk given by different cows are found as follows;**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Cow**  **Diet** | **I** | **II** | **III** | **IV** | **V** | **VI** | **VII** | **VIII** | **IX** |
| **D1** | **I** | **I** | **D** | **I** | **D** | **I** | **I** | **D** | **I** |
| **D2** | **D** | **D** | **I** | **D** | **I** | **D** | **D** | **I** | **I** |
| **D3** | **I** | **D** | **I** | **D** | **D** | **I** | **I** | **D** | **D** |
| **D4** | **I** | **I** | **I** | **D** | **D** | **I** | **I** | **D** | **I** |

**Test whether diets are equally effective or not at 1% level of significance.**

Working formula

Problem to test

H0: All the diets are equally effective

H1: All the diets are not equally effective.

Sum all the I(Positive) according to treatment to get Ri (Row wise) and according to objects to get Ci (Column wise) , i = 1 , 2 , 3 , …… k and j = 1, 2, 3, …… n. Then get

Ri, Ri2 Cj, Cj2

Test statistic

Q =

Critical value

2α (k -1) is the critical value obtained according to level of significance α and degree of freedom.

Decision

Reject H0 at α% level of significance if Q > 2α(k -1 ), accept otherwise.

1. **Following data represent the operating times in hours for four types of laptop before a charge is required.**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Dell** | **5.3** | **4.8** | **6.1** | **3.5** |  |  |  |
| **Acer** | **5.2** | **5.8** | **3.9** | **4.6** | **4.9** | **5.1** | **5.6** |
| **HP** | **4.5** | **5.2** | **3.8** | **4.8** | **5.3** |  |  |
| **Lenovo** | **4.7** | **6.2** | **5.7** | **5.5** | **3.9** | **4.8** |  |

**Are operating time for all laptops equal at 5% level of significance use non parametric test?**

Working formula

Problem to test

H0: Md1 = Md2 = Md3 = …………… = Mdk

H1: At least one Mdi is different i = 1, 2, 3, ………..k.

Combine n1, n2, n3 ……… and nk such that n1 + n2 + n3 + ……… + nk = n and rank these n observations in ascending order .If two or more observations are equal then assign average rank and is called tied. Sum the ranks of sample of sizes n1, n2, n3, ……… and nk separately to get R1, R2, R3,……… Rk.

Test statistic

H = Σ – 3(n + 1) ~ 2(k – 1)

If tied occurs then corrected test statistic is

H =ΣΣ, tᵢ = number of times ith rank is repeated.

Critical value

For ni < 5 and k = 3, critical value p is obtained from Kruskal Wallis table.

For ni > 5 and k > 3, critical value isχ²α (k -1).

Decision

Accept H0 at α level of significance if p > α, reject otherwise for ni < 5 and k=3.

Reject H0 at α level of significance if H > χ²α (k–1), accept otherwise for ni ˃ 5 and k ˃ 3.

1. **The scores of 7 students in Statistics II in three test are found as**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Student**  **Test** | **A** | **B** | **C** | **D** | **E** | **F** | **G** |
| **I** | **15** | **13** | **8** | **12** | **9** | **16** | **13** |
| **II** | **14** | **16** | **12** | **10** | **14** | **11** | **6** |
| **III** | **10** | **12** | **5** | **16** | **8** | **14** | **16** |

1. **Is there any significant difference in marks in three test?**
2. **Is there any significant difference in marks of seven students?**

**Use non parametric test at 1% level of significance.**

Working formula

Problem to test

H0: Md1 = Md2 = Md3 = …………… = Mdk

H1: At least one Mdi is different i = 1, 2, 3, ………. k.

Rank k sample observations for each block separately from 1 to k. in ascending order. If two or more observations are same then assign average rank which is also called tied. Obtain sum of ranks for each sample to get Ri, i = 1, 2, 3 , ………k

Test statistic

Fr = Ri2 – 3n(k + 1)

If tied occurs then corrected test statistic is

Fr = , tᵢ = number of times i th rank is repeated.

Critical value

For 2 < n < 9 and k = 3, also 2 < n < 4 and k = 4 critical value p is obtained from Friedman probability table.

For n > 5 and k > 3, critical value is χ²α (k – 1)

Decision

Accept H0 at α level of significance if p > α, reject otherwise for 2 < n < 9 and k = 3, also   
2 < n < 4 and k = 4.

Reject H0 at α level of significance if H > χ²α (k–1), accept otherwise for n > 5 and k ˃ 3.

1. **Let A, H, D and L represents Acer, HP, Dell and Lenovo laptop and following information represents their operating time in hours before charge is required.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **A**  **5.2** | **H**  **3.8** | **D**  **4.6** | **H**  **5.2** | **D**  **3.6** | **L**  **4.4** |
| **L**  **5.6** | **A**  **3.9** | **H**  **4.6** | **L**  **6.2** | **L**  **4.8** | **A**  **3.5** |
| **H**  **4.4** | **D**  **3.6** | **L**  **5.2** | **D**  **4.8** | **A**  **4.2** | **D**  **5.4** |
| **A**  **6.1** | **L**  **4.7** | **A**  **3.2** | **H**  **5.3** | **D**  **4.8** | **H**  **3.9** |

**Carryout analysis of the design at 1% level of significance.**

Working formula

Mathematical model

yij = μ + τᵢ + eij ;i = 1,2,3,………,t; j = 1,2,3, ………, r

Problem to test

H0T: μ1. = μ2. = μ3. =…. = μt. (There is no significant difference between the treatments)

H1T: At least one μᵢ. is different. i = 1, 2, 3……. t (There is at least one significant difference between treatments)

Test statistic

FT = MST/MSE , where MST = SST/(t – 1) , MSE = SSE / t(r – 1)

TSS = yij2 – C.F.

SST = – C.F. where C.F. =

SSE = TSS – SST

Critical value

At α level of significance critical value is Fα {(t–1), t(r–1)}

Decision

Reject at α level of significance if FT > Fα {(t–1), t(r–1)} accept otherwise.

1. **Let A, H, D and L represents Acer, HP, Dell and Lenovo laptop and following information represents their operating time in hours before charge is required.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **A**  **5.0** | **H**  **3.6** | **D**  **4.8** | **A**  **4.2** | **D**  **3.8** | **L**  **4.6** |
| **L**  **5.4** | **A**  **4.9** | **H**  **4.3** | **L**  **5.2** | **L**  **5.8** | **A**  **5.5** |
| **H**  **4.8** | **D**  **4.6** | **L**  **5.5** | **D**  **4.6** | **A**  **5.2** | **D**  **5.0** |
| **D**  **6.0** | **L**  **4.5** | **A**  **3.9** | **H**  **5.1** | **H**  **4.9** | **H**  **4.9** |

**Carryout analysis of the design at 1% level of significance.**

Working formula

Mathematical model

Yij = μ + τᵢ + βj + eij

Where

yij = jth block receiving ith treatment.

i = 1, 2, ……., t, j = 1, 2, …….., b

Problem to test

Problem to test

H0T: μ1. = μ2. = μ3. =…. = μt. (There is no significant difference between the treatments)

H1T: At least one μᵢ. is different. i = 1, 2, 3……. t (There is at least one significant difference between treatments)

H0B: μ.1 = μ.2 = μ3. =…. = μ.b (There is no significant difference between blocks)

H1T: At least one μᵢ. is different. j = 1, 2, 3……. b (There is at least one significant difference between blocks)

Test statistic

FT = MST/MSE

FB = MSB/MSE , where MST = SST/(t – 1) , MSB = SSB/(b – 1) MSE = SSE / (t-1)(b – 1)

TSS =

SST = – C.F. where C.F. =

SSB = – C.F.

SSE = TSS – SST - SSB

Critical value

At α level of significance critical value is Fα {(t–1), (t-1)(r–1)} for treatment and Fα {(r–1), (t-1)(r–1)} for block

Decision

Reject at α level of significance if FT > Fα {(t–1), (t-1)(b–1)} accept otherwise.

Reject at α level of significance if FB > Fα {(b–1), (t-1)(b–1)} accept otherwise.

1. **Let A, H, D and L represents Acer, HP, Dell and Lenovo laptop and following information represents their operating time in hours before charge is required.**

|  |  |  |  |
| --- | --- | --- | --- |
| **A**  **4.2** | **H**  **4.8** | **D**  **4.2** | **L**  **6.2** |
| **L**  **4.6** | **A**  **5.9** | **H**  **4.8** | **D**  **5.2** |
| **H**  **5.4** | **D**  **5.6** | **L**  **5.6** | **A**  **4.8** |
| **D**  **4.1** | **L**  **5.7** | **A**  **4.2** | **H**  **4.3** |

**Carryout analysis of the design at 5% level of significance.**

Working formula

Mathematical model

Yijk = μ + αᵢ + βj + eijk

Where

yijk = ith row and jth column receiving kth treatment.

i = 1, 2, ……., m, j = 1, 2, …….., m, k = 1,2,……….m

Problem to test

H0R: μ1.. = μ2.. = μ3.. =…. = μm.. (There is no significant difference between rows)

H1R: At least one μᵢ. is different. i = 1, 2, 3……. m (There is at least one significant difference between rows)

H0C: μ.1. = μ.2.= μ3. =…. = μ.m. (There is no significant difference between columns)

H1C: At least one μᵢ. is different. j = 1, 2, 3……. m (There is at least one significant difference between columns)

H0T: μ..1 = μ..2= μ . .3 =…. = μ. .m (There is no significant difference between treatments)

H1T: At least one μᵢ. is different. k = 1, 2, 3……. m (There is at least one significant difference between treatments)

Test statistic

FR = MSR/MSE

FC = MSC/MSE

FT = MST/MSE , where MSR = SSR/(m – 1) , MSC = SSC/(m – 1) , MST = SST/(m – 1)

MSE = SSE / (m-1)(m-2)

TSS = - CF

SSR = – C.F.

SSB = – C.F.

SST = – C.F.

where C.F. =

SSE = TSS – SSR – SSC - SST

Critical value

At α level of significance critical value is Fα {(m–1), (m-1)(m–2)}

Decision

Reject at α level of significance if F\* > Fα {(m–1), (m-1)(m–2)} accept otherwise.

Where \* = R,C,T